BASIC APPLIED MATHEMATICS

FORM SIX 2022

MARKING GUIDE

1. (a) (i) 7.0×10^{12}
   (ii) 1.61
   (iii) 0.15810

   (b) $Mean = (\overline{x}) = 40.16$
   S.D = 16.83

2. (a) Given
   $f(x) = x^2 + 2, g(x) = \sqrt{x - 4}$
   $gof(x) = \sqrt{x^2 + 2 - 4}$
   $= \sqrt{x^2 - 2}$
   $gof(\sqrt{11}) = \sqrt{(\sqrt{11})^2 - 2}$
   $= \sqrt{11 - 2}$
   $= 9$
   $\boldsymbol{gof(\sqrt{11}) = 3}$

   (b) $f(x) = \frac{x^2+2x-3}{x^2-16}$

   V.A : $x^2 - 16 = 0$

   $x = \pm 4$

   $H.A : Y = 1$

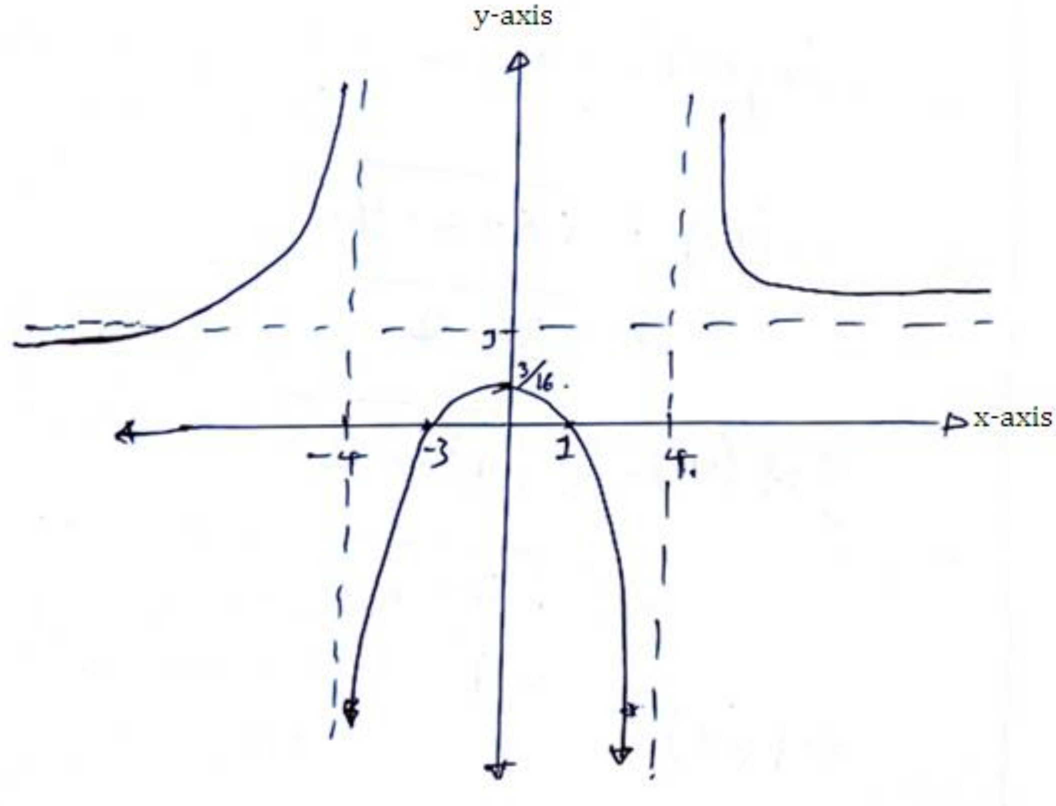
   $x - intercept\ when\ y = 0$

   $x^2 + 2x - 3 = 0$

   $x = -3 \quad x = 1$

   $y - intercept\ when\ x = 0$

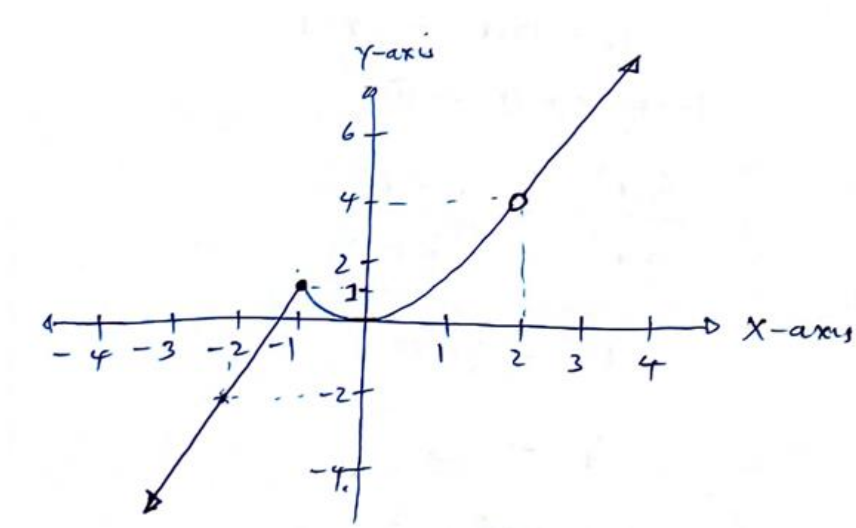
   $y = {}^{3}/_{16} = 0.1875 \approx 0.2$



Domain: $\{x: x \in R,\ x \neq \pm 4\}$
Range: $\{y: y \in R\ \ y \leq {}^{3}/_{16}\ and\ y > 0.9$

(c) Given $f(x) = \begin{cases} 3x + 4\ if\ x \leq -1 \\ x^2\ if -1 < x \leq 2 \\ 4x - 4\ if\ x > 2 \end{cases}$

(i) Graph



(ii) $f(-3) + f(2) + f(4)$

$f(-3) = 3 \times -3 + 4 = -5$

$f(2) = x^2 = (2)^2 = 4$

$f(4) = 4x - 4 = 4(4) - 4 = 12$

$f(-3) + f(2) + f(4) = -5 + 4 + 12$

$\underline{\underline{= 11}}$

3. (a) $G_3 = 27, \ G_6 = 729$

(i) But $G_3 = G_1 r^2 = 27 \ldots\ldots\ldots (i)$

$G6 = G_1 r^5 = 729 \ldots\ldots\ldots..(ii)$

Taking $eqn \ (ii) \div (i)$

$$\frac{G_1 r^5}{G_1 r^2} = \frac{729}{27}$$

$$\sqrt[3]{r^3} = \sqrt[3]{27}$$

$r = 3$

$\underline{\underline{\text{Common ratio = 3}}}$

(ii) Sum of first 8 terms

$$S_n = \frac{G_1(r^n - 1)}{r - 1}$$

$n = 8, \ G_1 = 3. \ r = 3$

$$S_8 = \frac{3(3^8 - 1)}{3 - 1}$$

$S_8 = 9840$

$\underline{\underline{\text{Sum of the first 8 terms of G.P = 9840}}}$

(b) $Z \propto \frac{y^3}{Z^2}$ Given $x = 8, \ y = 4, \ Z = 6$

$$X = \frac{K\,y^3}{Z^2}$$

$$8 = \frac{K(4)^3}{6^2}$$

$$K = \frac{8 \times 6^2}{4^3} = \frac{\cancel{8 \times 36}}{\cancel{64}} = {}^9/_2$$

$$K = {}^9/_2$$

Now: Finding x when $y = 2, Z = 12$

$$X = \frac{Ky^3}{Z^2} = \frac{{}^9/_2 \times 2^3}{12^2}$$

$\boldsymbol{X = {}^{1}/_{4}}$

(c) (i) Given $-8-4-2-1+\cdots$

It is a G.P

From $G_n = G_1 r^{n-1}$

$G_1 = -8; r = {}^{-1}/_{-8} = {}^{1}/_{2}$

$G_n = (-8)\left({}^{1}/_{2}\right)^{n-1}$

OR

$G_n = -8(2)^{1-n}$

OR

$G_n = -2^{4-n}$

$\therefore -8-4-2-1 \ldots\ldots\ldots - 8(2)^{1-n}$

In sigma notation

$= \boldsymbol{\sum_{r=1}^{n} -8(2)^{1-r}}$

(ii) 1 U.S dollar = 4000Tshs.

1000000U.s = x

$x = \frac{4000Tsh \times 1{,}000{,}000\ dollar}{1\ dollar}$

$x = 4{,}000{,}000Tsh$

$1000000dollars = 4{,}000{,}000Tshs$

$Again: If\ 1\ Euro = 2000\ Tsh$

$x?\ Euro = 4{,}000{,}000{,}000$

$x = \frac{4000000000}{2000}$

$x = 2000{,}000\ Euro$

$\therefore$ ***2,000,000 Euro will be obtained.***

4. (a) (i) $XY^2 + X^2Y - 5X + 3Y = 0$

$$X\left(2y\, {}^{dy}/_{dx}\right) + y^2 + x^2\frac{dy}{dx} + 2yx - 5 + 3\frac{dy}{dx} = 0$$

$$\frac{dy}{dx}(2xy + x^2 + 3) = 5 - y^2 - 2xy$$

$$\boldsymbol{{}^{dy}/_{dx} = \frac{5-y^2-2xy}{2xy+x^2+3}}$$

(ii) $y = \frac{e^x Cosx}{X^3}$

$$\frac{dy}{dx} = \frac{X^3(e^x(-sinx)+e^x CosX) - e^x CosX\ 3x^2}{X^6}$$

$$\frac{dy}{dx} = \frac{X^3e^X CosX - X^3e^X SinX - 3e^x X^2 CosX}{X^6}$$

$$\boldsymbol{{}^{dy}/_{dx} = \frac{e^X(XCosx - XsinX - 3CosX)}{X^4}}$$

(b) $f(x) = \sqrt{x}$

$f(X+h) = \sqrt{X+h}$

From ; $f'(x) = lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$

$= lim_{h\to 0}\frac{\sqrt{x+h}-\sqrt{x}}{h}$

Rationalizing the numerator

$$\frac{\sqrt{x+h}-\sqrt{x}}{h} \times \frac{\sqrt{x+h}+\sqrt{x}}{\sqrt{x+h}+\sqrt{x}}$$

$$= \frac{x+h-x}{h(\sqrt{x+h}+\sqrt{x})}$$

$$\lim_{h\to 0}\frac{\cancel{h}}{\cancel{h}(\sqrt{x+h})+\sqrt{x}}$$

$$= as\ h\to 0\ \frac{1}{\sqrt{x}+\sqrt{x}} = \frac{1}{2\sqrt{x}}$$

$$\boldsymbol{f'(x) = \frac{1}{2\sqrt{x}}}$$

(c) Given

$radius\ (r) = 3cm$

$\frac{dr}{dt} = 2cm/s$

$Required:\ \frac{dA}{dt} = ?$

From $A = 4\pi r^2$

$\frac{dA}{dr} = 8\pi r$

$But: \frac{dA}{dt} = \frac{dA}{dr}\frac{dr}{dt}$

$= 8\pi r.2$

$= 16\pi r \quad and\ r = 3$

$= 16\pi \times 3$

$\frac{dA}{dt} = 48\pi\ cm^2/s$

$\underline{Rate\ of\ change\ of\ area = 48\pi cm^2/s}$

5. (a) (i) $\int \left(\sqrt{x} + \frac{1}{\sqrt{x}}\right) dx$

$= \int \sqrt{x}dx + \int \frac{1}{\sqrt{x}} dx$

$= \int x^{1/2}dx + \int X^{-1/2}dx$

$= \frac{x^{3/2}}{3/2} + \frac{X^{1/2}}{1/2} + C$

$= \mathbf{\frac{2}{3}X^{3/2} + 2\sqrt{X} + C}$

(ii) $\int cos^2X\ SinXdx$

$let\ u = Cosx$

$\frac{du}{dx} = -Sinx$

$dx = \frac{du}{-Sinx}$

$\int Cos^2 \times sinxdx = \int u^2 \cancel{Sinx} \frac{du}{\cancel{-sinx}}$

$= -\int u^2du = \frac{-u^3}{3} + c$

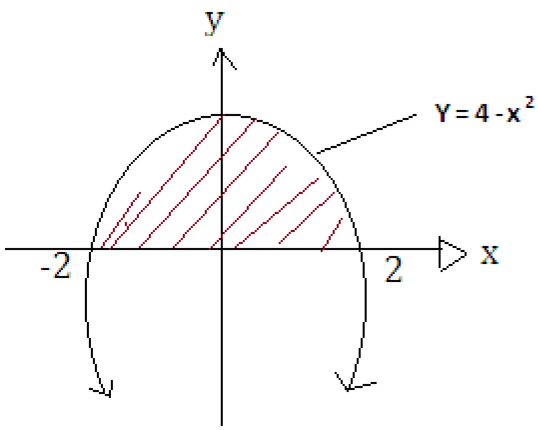
$\int \mathbf{Cos^2xSinxdx} = \frac{-u^3}{3} + c = \frac{-cos^3x}{3} + C$

(b) $y = 4 - x^2, x - axis\ y = 0$

$0 = 4 - x^2$

$x^2 = 4, x = \pm 2$

Sketch



$A = \int_a^b f(x)dx$

$A = \int_{-2}^{2}(4 - x^2)dx$

$= 4x - \frac{x^3}{3}\Big]_{-2}^{2}$

$\boldsymbol{A = \left(8 - \frac{8}{3}\right) - \left(-8 + \frac{8}{3}\right) = \frac{32}{3}}$

(c) Given $MR(x) = 375 - X - 3X^2$

From $\frac{d}{dx}(Toal\ revenue) = maginal\ revenue$

$T.R = \int M.R.dx$

$T.R = \int_{20}^{30}(375 - x - 0.5x^2)dx$

$= 375x - \frac{x^2}{2} - \frac{0.5x^3}{3}\Big]_{20}^{30}$

$= \left(375 \times 30 - \frac{30^2}{3} - \frac{0.5 \times 30^3}{3}\right) - \left(375 \times 20 - \frac{20^2}{2} - \frac{0.5\ 20^3}{3}\right)$

$T.R = 6300 - \frac{17900}{3}$

$\underline{\boldsymbol{T.R = 333.33\ units}}$

6. Frequency distribution table

C. interval	freq	$C.m(x)$	fx	fx^2
21 – 30	3	25.5	76.5	1950.75
31 – 40	T	35.5	213	7561.5
41 – 50	T+4	45.5	455	20702.5
51 – 60	8	55.5	444	24642
61 – 70	6	65.5	393	25741.5
71 – 80	4	75.5	302	22801
81 - 90	3	85.5	256.5	21930.75
	Ʃ =40		Ʃ =2140	Ʃ =125330

(i) Value of t

Total student = 40

$3 + t + t + 4 + 8 + 6 + 4 + 3 = 40$

$2t = 12$

$\underline{\boldsymbol{t = 6}}$

(ii) $Mean\ (\overline{x}) = \frac{\varepsilon f(x)}{\varepsilon f}$

$\overline{x} = \frac{2140}{40}$

$\underline{\boldsymbol{\overline{x} = 53.5}}$

$Median = L + \left(\frac{N/_2 - nb}{nw}\right) C$

$L = 50.5 \qquad C = 10$

$nb = 19$

$nw = 8$

$Median = 50.5 + \left(\frac{\frac{40}{2} - 19}{8}\right) 10$

$\underline{\boldsymbol{Median = 51.75}}$

(iii) $S.D = \sqrt{var(x)}$

$= \sqrt{\frac{\varepsilon f x^2}{\varepsilon f} - \left[\frac{\varepsilon f(x)}{\varepsilon f}\right]^2}$

$= \sqrt{\frac{125330}{40} - \left[\frac{2140}{40}\right]^2}$

$S.D = \sqrt{271}$

$\underline{\boldsymbol{S.D = 16.462}}$

7. (a) $p(n,4) = 42p(n,2)$

$From\ p|(n,r) = \frac{n!}{(n-2)!}$

$\frac{\cancel{n!}}{(n-4)!} = 42\left(\frac{\cancel{n!}}{(n-21)!}\right)$

$(n-2)! = 42(n-4)!$

$(n-2)(n-3)(n-4)! = 42(n-4)!$

$(n-2)(n-3) = 42$

$n^2 - 5n + 6 = 42$

$n^2 - 5n - 36 = 0$

$n = 9,\ n = -4$

$Ignore\ n = -4$

$\underline{\boldsymbol{n = 9}}$

(b) $Given\ P(A) = {}^{1}/_{4},\ P(B) = {}^{1}/_{2},\ P(AnB) = {}^{1}/_{8}$

(i) $P(AuB') = P(A) + p(B') - p(AnB')$

$But\ p(B') = 1 - p(B)$

$p(AnB') = p(A) - p(AnB)$

$p(AuB') = p(A) + 1 - p(b) - [p(A) - p(AnB)].$

$= {}^{1}/_{4} + 1 - {}^{1}/_{2} - ({}^{1}/_{4} - {}^{1}/_{8})$

$p(AuB') = {}^{5}/_{8}$

(ii) $p({}^{A}/_{B}) = \frac{p(AnB)}{p(B)}$

$= \frac{\frac{1}{8}}{{}^{1}/_{2}}$

$\boldsymbol{p({}^{A}/_{B}) = {}^{1}/_{4}}$

(c) Required 5 digits telephone number from 0 to 9

Numbers 0, 1, 2, 3, 4, 5, 6, 7, 8, 9

Number	1st	2nd	3rd	7	5
Ways	8	7	6	1	1

Total ways : $8 \times 7 \times 6 \times 1 \times 1$

$\underline{\boldsymbol{= 336\ ways}}$

8. (a) $|Sin\ A = {}^{4}/_{5} \quad Cos\ B = {}^{12}/_{13}$

$Cos(A + B) = CosACosB - SinASinB$

$But\ CosA = \sqrt{1 - Sin^2 A}$

$CosA = \sqrt{1 - \left({}^{4}/_{5}\right)^2} = \frac{3}{5} \ldots..(i)$

$SinB = \sqrt{1 - Cos^2 B}$

$= \sqrt{1 - \left(\frac{12}{13}\right)^2} = \frac{5}{13} \ldots\ldots..(ii)$

$From; Cos(A + B) = CosACosB - SinASinB.$

$= \frac{3}{5} \times \frac{12}{13} - \frac{4}{5} \times \frac{5}{13}$

$= \frac{36-20}{65}$

$\boldsymbol{Cos(A + B) = \frac{16}{65}}$

(b) $If\ a = sinx,\ b = CosX$

$a\left(\sqrt{1-b^2}\right) + b\left(\sqrt{1-a^2}\right)$
$= sinx\left(\sqrt{1-Cos^2x}\right) + cosx\sqrt{1-Sin^2x}$
$= Sin^2x + Cos^2x$
$\underline{\mathbf{= 1}}$

(c) $4Cos\theta - 3sec\theta = 2tan\theta \;\; for\ 180^{\circ} \leq \theta \leq 180^{\circ}$
Solution
$4cos\theta - 3sec\theta = 2tan\theta$
$4cos\theta - \frac{3}{cos\theta} = \frac{2sin\theta}{cos\theta}$
$4cos^2\theta - 3 = 2sin\theta$
$But\ Cos^2\theta = 1 - Sin^2\theta$
$4(1 - sin^2\theta) - 3 = 2sin\theta$
$4 - Sin^2\theta - 3 = 2sin\theta$
$4Sin^2\theta + 2sin\theta - 1 = 0$

$Solving\ quadratically$
$sin\theta = \frac{-1+\sqrt{5}}{4}\ or\ \frac{-1-\sqrt{5}}{4}$
$\theta = Sin^{-1}\left(\frac{-1+\sqrt{5}}{4}\right) = or\theta = Sin^{-1}\left(\frac{-1-\sqrt{5}}{4}\right)$
$\theta = 18^{\circ} = \propto$
$\propto = -54^{\circ}$

General formula for sine
$\theta = \pi n + (-1)^n \propto$
$for\ \ \propto = 18^{\circ}$
$\theta = 180n + (-1)^n 18$
$\theta = 18^{\circ},\ 162^{\circ}$
$for\ \ \propto = -54^{\circ}$
$\theta = 180n + (-1)^n \times (-54)$
$\theta = -54^{\circ}$
$\underline{: \boldsymbol{\theta} = \mathbf{-54^{\circ}, 18^{\circ}, 162^{\circ}}}$

9. (a) Given

$$A = \begin{bmatrix} 2 & 3 & 1 \\ 2 & 4 & 1 \\ 2 & 3 & 4 \end{bmatrix}\ and\ B = \begin{bmatrix} 2 & 1 & 3 \\ 1 & 5 & 0 \\ 2 & 4 & 2 \end{bmatrix}$$

$Find\ 3A - 2B$

$$= \left(3\begin{bmatrix} 2 & 3 & 1 \\ 2 & 4 & 1 \\ 2 & 3 & 4 \end{bmatrix} - 2\begin{bmatrix} 2 & 1 & 3 \\ 1 & 5 & 0 \\ 2 & 4 & 2 \end{bmatrix} \right)$$

$$= \left(\begin{bmatrix} 6 & 9 & 3 \\ 6 & 12 & 3 \\ 6 & 9 & 12 \end{bmatrix} - \begin{bmatrix} 4 & 2 & 6 \\ 2 & 10 & 0 \\ 4 & 8 & 4 \end{bmatrix} \right)$$

$$3A - 2B = \begin{bmatrix} 2 & 7 & -3 \\ 4 & 2 & 3 \\ 2 & 1 & 8 \end{bmatrix}$$

(b)

(i) Required : To find inverse of matrix of $\begin{bmatrix} 1 & 2 & 3 \\ 2 & 1 & 1 \\ 3 & 1 & -2 \end{bmatrix}$

Let m be $\begin{bmatrix} 1 & 2 & 3 \\ 2 & 1 & 1 \\ 3 & 1 & -2 \end{bmatrix}$

$$|m| = 1\begin{vmatrix} 1 & 1 \\ 1 & -2 \end{vmatrix} - 2\begin{vmatrix} 2 & 1 \\ 3 & -2 \end{vmatrix} - 3\begin{vmatrix} 2 & 1 \\ 3 & 1 \end{vmatrix}$$

$= 1(-2-1) - 2(-4-3) + 3(2-3)$

$= 8$

$\underline{Cofactor\ of\ m}$

$1: +(-2-1) = -3$

$2: -(-4-3) = 7$

$3: +(2-3) = -1$

$2: -(-4-3) = 7$

$1: +(-1-9) = -11$

$1: -(1-6) = 5$

$3: +(2-3) = -1$

$1: -(1-6) = 5$

$-2: +(1-4) = -3$

$$Cofactor\ of\ matrix\ M = \begin{bmatrix} -3 & 7 & -1 \\ 7 & -11 & 5 \\ -1 & 5 & -3 \end{bmatrix}$$

$$M^T = \begin{bmatrix} -3 & 7 & -1 \\ 7 & -11 & 5 \\ -1 & 5 & -3 \end{bmatrix}$$

$$M^{-1} = \frac{1}{|M|} \times Adj\ M$$

$$= \frac{1}{8}\begin{bmatrix} -3 & 7 & -1 \\ 7 & -11 & 5 \\ -1 & 5 & -3 \end{bmatrix}$$

$$m^{-1} = \begin{bmatrix} -3/8 & 7/8 & -1/8 \\ 7/8 & -11/8 & 5/8 \\ -1/8 & 5/8 & -3/8 \end{bmatrix}$$

(iii) Required : To solve for $x, y \ and \ z$ using inverse method

$$x + 2y + 3z = 6$$

$$2x + y + z = 5$$

$$3x + y - 2z = 1$$

In matrix form

$$\begin{bmatrix} 1 & 2 & 3 \\ 2 & 1 & 1 \\ 3 & 1 & -2 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 6 \\ 5 \\ 1 \end{bmatrix} \ldots \ldots .(i)$$

Multiply by M^{-1} both sides in eqn 1

$$M^{-1} \begin{bmatrix} 1 & 2 & 3 \\ 2 & 1 & 1 \\ 3 & 1 & -2 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = M^{-1} \begin{bmatrix} 6 \\ 5 \\ 1 \end{bmatrix}$$

But from the properties of matrix $M^{-1}M = I$ (*identity matrix*

$MI = M$ where I= identity matrix

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -3/8 & 7/8 & -1/8 \\ 7/8 & -11/8 & 5/8 \\ -1/8 & 5/8 & -3/8 \end{bmatrix} \begin{bmatrix} 6 \\ 5 \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 2 \\ -1 \\ 2 \end{bmatrix}$$

$\boldsymbol{x = 2, \ y = -1 \ and \ z = 2}$

10. (a) (i) **LINEAR PROGRAMMING**

Is the process of maximizing or minimizing the linear functions under linear inequalities constraints

(ii) **FEASIBLE REGION**

Is the region of the graph containing all the points that satisfy all the inequalities in a system.

(b) Let x be number of buses

y be number of Bajaji

	Bus	**Bajaji**	**Total**
Units	3	1	54
Cost	130000	72,000	

$x \geq 5, y \geq 10$

$x + y \leq 30$

$3x + y \leq 54$

$x \geq 0$

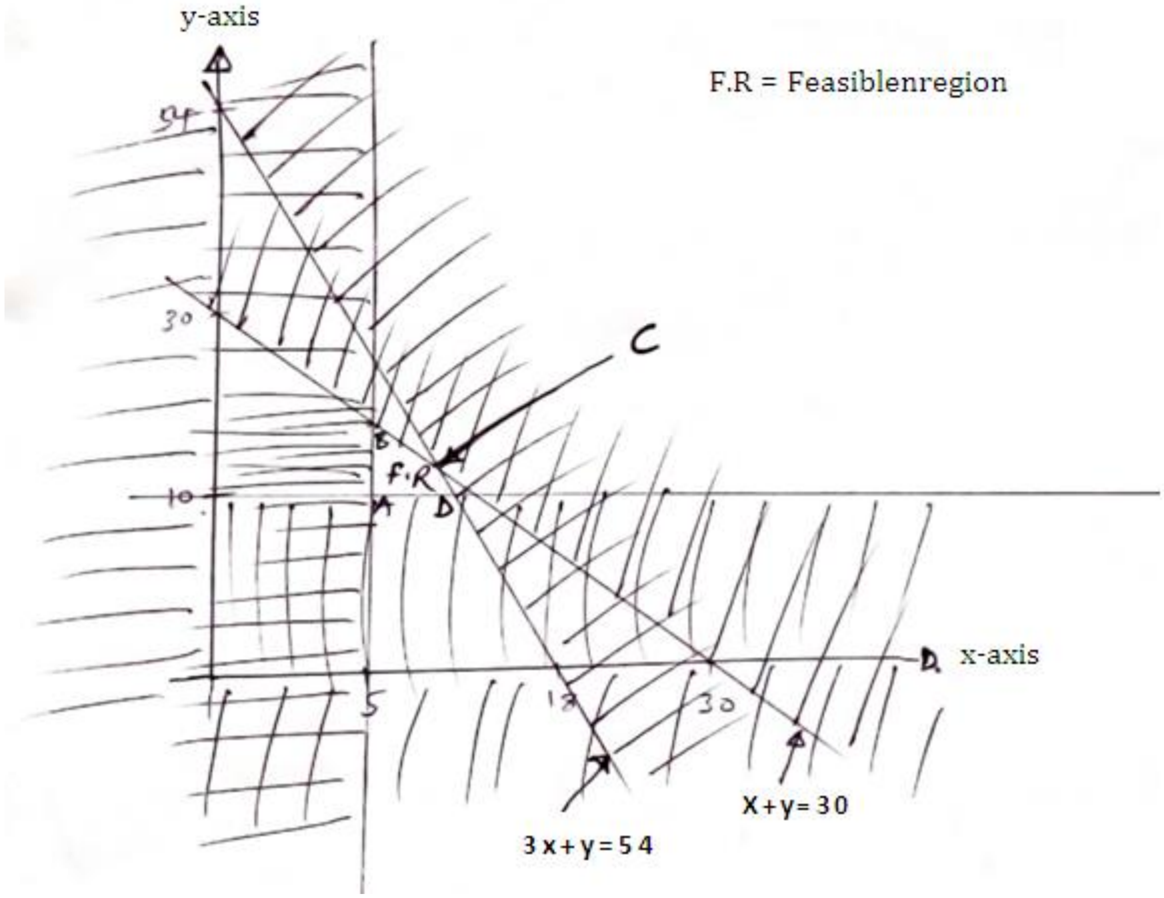
$y \geq 0$

Objective function;

$f(x, y) = 135,000x + 72000y$

Corner points	Cost $f(x, y) = 135000x + 72000y$
A (5,10)	1,395,000/=
B (5,25)	2,475,000/=
C (12, 18)	2,916,000/=
D (14, 10)	2,610,000/=

Graph



Maximum cost is 2,628,000Tsh and corresponding number of buses is 12 and Bajaji is 14