

MARKING SCHEME

1. (a) Solution

$$P(A \cap B) = 0.7 \text{ and } P(A \cup B) = 0.2$$

$$\text{From } P(A \cup B) = (P(A) + P(B) - P(A \cap B))$$

$$0.7 = P(A) + P(B) - 0.2$$

$$P(A) + P(B) = 0.9 \dots\dots\dots(i) \quad (1 \text{ Mark})$$

But

$$P(A) + P(A') = 1 \dots\dots\dots(ii)$$

$$P(B) + P(B') = 1 \dots\dots\dots(iii) \quad (1 \text{ Mark})$$

Add equations (i) and (ii)

$$P(A) + P(B) + P(A') + P(B') = 2 \dots\dots\dots(iv)$$

Substitute equation (i) into equation (iv)

$$0.9 + P(A') + P(B') = 2$$

$$P(A') + P(B') = 1.1.$$

(b) Given that $p(x) = 0.5 - rx$ in the interval $0 < x < 4$

(i) $r = ?$

$$\text{From } \int_0^4 p(x) dx = 1 \quad (0.5 \text{ Mark})$$

$$\text{From } \int_0^4 (0.5 - rx) dx = 0.5(4 - 16r) = 1 \quad (1 \text{ Mark})$$

$$\text{The value of } r = \frac{1}{8}$$

(ii) $E(x) = ?$

$$\text{From } E(x) = \int_0^4 xP(x) dx$$

$$E(x) = \int_0^4 x \left(\frac{1}{2} - \frac{1}{8}x \right) dx = \left[\frac{1}{4}x^2 - \frac{1}{24}x^3 \right]_0^4 = \frac{4}{3}$$

The expectation of x is $\frac{4}{3}$ (1 Mark)

(i) $\text{Var}(x) = ?$

$$\text{From } \text{Var}(x) = E(x^2) - [E(x)]^2$$

$$\text{Also } E(x^2) = \int_0^4 x^2 p(x) dx$$

$$E(x^2) = \int_0^2 x^2 \left(\frac{1}{2} - \frac{1}{8}x\right) dx = \int_0^2 \left(\frac{1}{2}x^2 - \frac{1}{8}x^3\right) dx = \left[\frac{1}{6}x^3 - \frac{1}{32}x^4\right]_0^2 = \frac{8}{3} \quad (1\text{Mark})$$

$$\text{Var}(x) = \frac{8}{3} - \left(\frac{3}{4}\right)^2 = \frac{8}{9}$$

Variance of x is $\frac{8}{9}$ (1Mark)

(ii) $P(1 < x < 2) = ?$

$$\text{From } P(1 < x < 2) = \int_1^2 p(x) dx \quad (0.5\text{Mark})$$

$$\int_1^2 \left(\frac{1}{2} - \frac{1}{8}x\right) dx = \left[\frac{1}{2}x - \frac{1}{16}x^2\right]_1^2 = \frac{5}{16}$$

$$P(1 < x < 2) = \frac{5}{16} \quad (1 \text{ Mark})$$

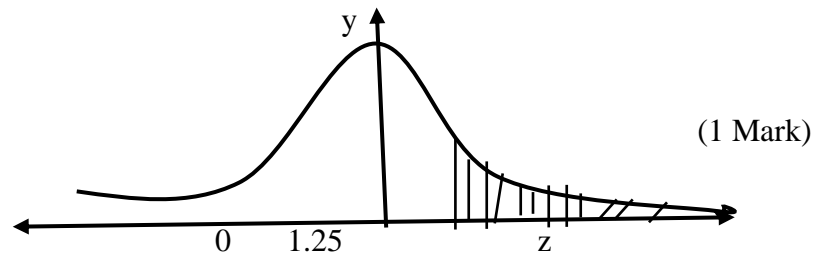
(c) Given, Mean = 20

Standard deviation = 4

$$\text{But, } Z = \frac{X - \mu}{\sigma} = \frac{X - 20}{4} \quad (0.5\text{Mark})$$

(i) $P(x > 25) = ?$

$$P(x > 25) = p\left(z = \frac{x-20}{4}\right) = p\left(z \geq \frac{25-20}{4}\right) = p(z > 1.25) \quad (0.5 \text{ Mark})$$



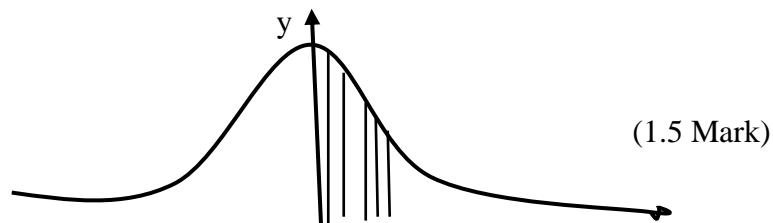
$$P(x > 25) = 0.1057$$

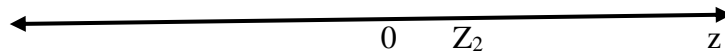
(ii) $d = ?$ Given that $p(20 < x < d) = 0.4641$

$$P\left(\frac{20-20}{4} < z < \frac{d-20}{4}\right) = 0.4641$$

$$P\left(0 < z < \frac{d-20}{4}\right) = 0.4641 \text{ let } Z_2 = \frac{d-20}{4} \quad (0.5\text{Mark})$$

$$P(0 < z < z_2) = 0.4641$$





by using z – scoresheet, $Z_2 = 1.8$

but $Z_2 = 1.8 = \frac{d - 20}{4}$

$d = 27.2$ (1 Mark)

2. (a) (i) Sentence is a basic unit of language that express a complete thought.

Example: Mbina is the most talented boy in the class.

While

Statement is a group of words which can be judged to be true or false.

Example: Mbina is the most talented boy. (1 Mark)

(ii) Use a truth table as follows

P	q	$p \rightarrow q$	$q \rightarrow p$	$p \leftrightarrow q$	$(p \rightarrow q) \wedge (q \rightarrow p)$
T	T	T	T	T	T
T	F	F	T	F	F
F	T	T	F	F	F
F	F		T	T	T

(2 Marks)

From the table above the column for $p \leftrightarrow q$ is the same as the column $(p \rightarrow q) \wedge (q \rightarrow p)$.

Hence, the two sentence are equivalent.

(b) Converse; “If they cancel school, then it rains” (1 Mark)

Contrapositive; “If they do not cancel school, then it does not rain.” (1 Mark)

Inverse “If it does not rain, then they do not cancel school.” (1 Mark)

(c) Proceed as follow

$[(p \rightarrow \sim q) \wedge (q \vee r) \wedge p] \rightarrow r$ Given

$\sim [(\sim p \vee \sim q) \wedge (q \vee r) \wedge p] \vee r$ by definition

$\sim [(\sim p \vee \sim q) \wedge p \wedge (q \vee r)] \vee r$ commutative law

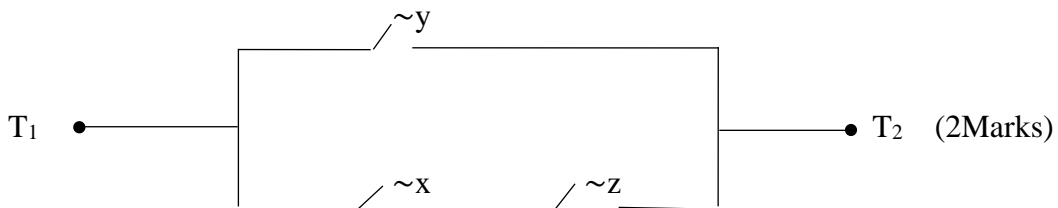
$\sim [(\sim p \wedge p) \vee (\sim q \wedge p) \wedge (q \vee r)] \vee r$ distributive law

$\sim [(F \vee (\sim q \wedge p) \wedge (q \vee r)] \vee r$ complement law

$\sim [(\sim q \wedge p) \wedge (qvr)]vr$ identify law
 $\sim (\sim q \wedge p) \vee \sim (qvr) vr$ Demorgan's law
 $(qv \sim p) \vee (\sim q \wedge \sim r)vr$ Demorgan's law
 $(qv \sim p) \vee [(\sim qvr) \wedge (\sim rvr)]$ Distributive law
 $(qv \sim p) \vee [(\sim qvr) \wedge T]$ Compliment law
 $(qv \sim p) \vee (\sim qvr)$ Identity law
 $(qv \sim p) \vee (\sim qvr)$ Complement law
 $T \vee (\sim pvr)$ Complement law
 T Identity law
 Hence the proposition is tautology (4Marks)

(d) Consider the rows containing F on the last column.

$[(\sim x \vee \sim y \vee \sim z) \wedge (\sim x \vee \sim y \vee z)] \wedge (x \vee \sim y \vee \sim z)$ Given
 $[(\sim x \vee \sim y) \vee (\sim z \wedge z)] \wedge (x \vee \sim y \vee \sim z)$ Distributive law
 $[(\sim x \vee \sim y) \vee F] \wedge (x \vee \sim y \vee \sim z)$ Complement law
 $(\sim x \vee \sim y) \wedge (x \vee \sim y \vee \sim z)$ Identity law
 $(\sim y \vee \sim x) \wedge (\sim y \vee x \vee \sim z)$ Commutative law
 $\sim y \vee [\sim x \wedge (x \vee \sim z)]$ Distributive law
 $\sim y \vee [(\sim x \wedge x) \vee (\sim x \wedge \sim z)]$ Distributive law
 $\sim y \vee [F \vee (\sim x \wedge \sim z)]$ Complement law
 $\sim y \vee (\sim x \wedge \sim z)$ Identity Law



3. (a) (i) Proceed as follow

Required a vector of 8 units in a direction of vector $5i - j + 2k$

Let the required vector = "b" in the direction $\hat{a} = 5i - j + 2k$

$b = 8a$ (i) (0.5 Mark)

$$\hat{a} = \frac{\underline{a}}{|\underline{a}|} = \frac{5\mathbf{i} - \mathbf{j} + 2\mathbf{k}}{\sqrt{5^2 + (-1)^2 + 2^2}} = \frac{5\mathbf{i} - \mathbf{j} + 2\mathbf{k}}{\sqrt{30}} \dots\dots\dots (ii) \quad (1 \text{ Mark})$$

substitute equation (ii) into equation (i)

$$\underline{b} = 8 \left(= \frac{5\mathbf{i} - \mathbf{j} + 2\mathbf{k}}{\sqrt{30}} \right) = \frac{8}{\sqrt{30}} (5\mathbf{i} - \mathbf{j} + 2\mathbf{k})$$

A vector is $\frac{8}{\sqrt{30}} (5\mathbf{i} - \mathbf{j} + 2\mathbf{k})$ (1Mark)

(ii) Proceed as follow

Let $\underline{a} = \mathbf{i} - 2\mathbf{j} + 3\mathbf{k}$, $|\underline{a}| = \sqrt{14}$

$\underline{b} = 3\mathbf{i} - 2\mathbf{j} + 3\mathbf{k}$, $|\underline{b}| = \sqrt{14}$

From $\underline{a} \cdot \underline{b} = |\underline{a}| |\underline{b}| \cos\theta$

$$\theta = \cos^{-1} \left(\frac{\underline{a} \cdot \underline{b}}{|\underline{a}| |\underline{b}|} \right) \dots\dots\dots (i) \quad (1 \text{ Mark})$$

$$\underline{a} \cdot \underline{b} = \begin{pmatrix} 1 \\ -2 \\ 3 \end{pmatrix} \cdot \begin{pmatrix} 3 \\ -2 \\ 1 \end{pmatrix} = 10 \dots\dots\dots(ii) \quad (1 \text{ Mark})$$

$$|\underline{a}| |\underline{b}| = (\sqrt{14})(\sqrt{14}) = 14 \dots\dots\dots (iii)$$

$$\theta = \cos^{-1} \left(\frac{10}{14} \right) = 44^{\circ}24'$$

The angle between the vectors is $44^{\circ}24' \approx 44.42^{\circ}$ (1 Mark)

(b) Proceed as follows

$\underline{a} = 3\mathbf{i} + \mathbf{j} - 2\mathbf{k}$, $\underline{b} = \mathbf{i} - 3\mathbf{j} - \mathbf{k}$ and $m:n = 1:3$

Internal division = $\left(\frac{m}{m+n} \right) \underline{b} + \left(\frac{n}{m+n} \right) \underline{a}$

Internal division = $\left(\frac{1}{1+3} \right) (\mathbf{i} - 3\mathbf{j} - \mathbf{k}) + \left(\frac{3}{1+3} \right) (3\mathbf{i} + \mathbf{j} - 2\mathbf{k})$

Internal division = $\frac{1}{4} (\mathbf{i} - 3\mathbf{j} - \mathbf{k}) + \frac{3}{4} (3\mathbf{i} + \mathbf{j} - 2\mathbf{k})$

Internal division = $\frac{1}{4} (\mathbf{i} - 3\mathbf{j} - \mathbf{k} + 9\mathbf{i} + 3\mathbf{j} - 6\mathbf{k})$

Internal division = $\frac{1}{4} (10\mathbf{i} - 7\mathbf{k})$

A position vector is $\frac{10}{4} \mathbf{i} - \frac{7}{4} \mathbf{k}$ (2 Marks)

(c) (i) $\lambda = ?$ when \underline{a} and \underline{b} are perpendicular.

$$\text{From } \underline{a} \cdot \underline{b} = |\underline{a}| |\underline{b}| \cos \theta$$

$$a \cdot b = |\underline{a}| |\underline{b}| \cos 90^\circ \quad (1 \text{ Mark})$$

$$a \cdot b = 0$$

$$a \cdot b = \begin{pmatrix} 2 \\ 4 \\ \lambda \end{pmatrix} \cdot \begin{pmatrix} 1 \\ 2 \\ 7 \end{pmatrix} = 0$$

$$2 + 8 + 7\lambda = 0$$

$$\lambda = -\frac{10}{7} \quad (1 \text{ Mark})$$

(ii) $\lambda = ?$ when \underline{a} and \underline{b} are collinear.

$$\text{From } \underline{a} \cdot \underline{b} = |\underline{a}| |\underline{b}| \cos \theta$$

$$\underline{a} \cdot \underline{b} = |\underline{a}| |\underline{b}| \cos \theta$$

$$\underline{a} \cdot \underline{b} = |\underline{a}| |\underline{b}| \dots \dots \dots (i) \quad (1 \text{ Mark})$$

$$a \cdot b = \begin{pmatrix} 2 \\ 4 \\ \lambda \end{pmatrix} \cdot \begin{pmatrix} 1 \\ 2 \\ 7 \end{pmatrix} = 10 + 7\lambda \dots \dots \dots (ii)$$

$$|\underline{a}| |\underline{b}| = (\sqrt{20 + \lambda^2})(\sqrt{54})$$

$$|\underline{a}| |\underline{b}| = (\sqrt{54}(20 + \lambda^2)) \dots \dots \dots (iii)$$

Substitute equation (ii) and (iii) into equation (i)

$$10 + 7\lambda = \sqrt{54(20 + \lambda^2)}$$

$$(10 + 7\lambda)^2 = 54(20 + \lambda^2)$$

$$100 + 140\lambda + 49\lambda^2 = 1080 + 54\lambda^2$$

$$5\lambda^2 - 140 + 980 = 0$$

$$\lambda = 14. \quad (1 \text{ Mark})$$

(d) Proceed

$$\text{Let } \underline{f}_m = 6\text{N}$$

$$= (1, -2, 2)$$

$$|\underline{f}| = \sqrt{1^2 + (-2)^2 + 2^2} = \sqrt{9} = 3$$

$$F = f_m \left(\frac{f}{|f|} \right) = 6 \left(\frac{i-2j+2k}{3} \right) = 2(i-2j + 2k)$$

$$F = 2i - 4j + 4k \dots\dots\dots (i) \quad (1 \text{ Mark})$$

The distance from a = (0,1,2) to b (-1,3,-2)

$$d = b - a = (-1,3,-2) - (0,1,2) = (-1,2,-4)$$

$$d = (-1,2, -4) \dots\dots\dots (ii) \quad (1 \text{ Mark})$$

From, work done = F.d

$$\text{Work done} = \begin{pmatrix} 2 \\ -4 \\ 4 \end{pmatrix} \cdot \begin{pmatrix} -1 \\ 2 \\ -4 \end{pmatrix} = -2-8 - 16 = |-26| = 26$$

The work done on moving a particle is 26 units. (1 Mark)

4. (a) Required to solve for z

Given that $z^3 = 1 + i\sqrt{3}$ express in polar form

$$|z| = r = 2, \text{ Arg } (z) = \tan^{-1} \left(\frac{\sqrt{3}}{1} \right) = \frac{\pi}{3}$$

$$\text{Now, } z^3 = 2 \left(\cos \frac{\pi}{3} + i \sin \frac{\pi}{3} \right)$$

$$\text{From } z_{k+1} = r^{\frac{1}{n}} \left[\cos \left(\frac{\theta+2\pi k}{n} \right) + i \sin \left(\frac{\theta+2\pi k}{n} \right) \right]$$

$$\text{From } z_{k+1} = r^{1/n} \left[\cos \left(\frac{\frac{\pi}{3}+2\pi k}{3} \right) + i \sin \left(\frac{\frac{\pi}{3}+2\pi k}{3} \right) \right]$$

$$\text{From } z_{k+1} = \sqrt[3]{2} \left[\cos \left(\frac{\pi+6\pi k}{9} \right) + i \sin \left(\frac{\pi+6\pi k}{9} \right) \right] \text{ where } k = 0,1,2$$

$$\text{When } k=0; Z_1 = \sqrt[3]{2} \left[\cos \left(\frac{\pi}{9} \right) + i \sin \left(\frac{\pi}{9} \right) \right] \quad (1 \text{ Mark})$$

$$\text{When } k=1; Z_2 = \sqrt[3]{2} \left[\cos \left(\frac{7\pi}{9} \right) + i \sin \left(\frac{7\pi}{9} \right) \right] \quad (1 \text{ Mark})$$

$$\text{When } k=2; Z_3 = \sqrt[3]{2} \left[\cos \left(\frac{13\pi}{9} \right) + i \sin \left(\frac{13\pi}{9} \right) \right] \quad (1 \text{ Mark})$$

(b) Required to show that; $\tan 3\theta = \frac{3t-t^3}{t-3t^3}$ where $t = \tan\theta$

$$\text{Consider } \tan 3\theta = \frac{\sin 3\theta}{\cos 3\theta} \dots\dots\dots (i)$$

First express $z = \cos 3\theta$ and $Z = \sin \theta$ into powered form

$$(\cos \theta + i \sin \theta)^3 = \cos 3\theta + i \sin 3\theta$$

$$\cos 3\theta + i \sin 3\theta = (\cos \theta + i \sin \theta)^3$$

$$\cos 3\theta + i \sin 3\theta = \cos^3 \theta + 3i \cos^2 \theta \sin \theta + 3i^2 \cos \theta \sin^2 \theta + i^3 \sin^3 \theta \text{ but } i^2 = -1, i^3 = -i$$

$$\cos 3\theta + i \sin 3\theta = \cos^3 \theta + 3i \cos^2 \theta \sin \theta - 3 \cos \theta \sin^2 \theta - i \sin^3 \theta$$

$$\cos 3\theta + i \sin 3\theta = (\cos^3 \theta - 3 \cos \theta \sin^2 \theta) + i(3 \cos^2 \theta \sin \theta - \sin^3 \theta) \dots\dots\dots (ii)$$

From Equation (ii) equate real part

$$\cos 3\theta = \cos^3 \theta - 3 \cos \theta \sin^2 \theta$$

$$\cos^3 \theta = \cos^3 \theta - 3 \cos \theta (1 - \cos^2 \theta)$$

$$\cos 3\theta = \cos^3 \theta - 3 \cos \theta + 3 \cos^3 \theta$$

$$\cos 3\theta = 4 \cos^3 \theta - 3 \cos \theta \dots\dots\dots (iii) \quad (1 \text{ Mark})$$

From equation (ii) equate imaginary part

$$\sin 3\theta = 3 \cos^2 \theta \sin \theta - \sin^3 \theta$$

$$\sin 3\theta = 3(1 - \sin^2 \theta) \sin \theta - \sin^3 \theta$$

$$\sin 3\theta = 3 \sin \theta - 3 \sin^3 \theta - \sin^3 \theta$$

$$\sin 3\theta = 3 \sin \theta - 4 \sin^3 \theta \dots\dots\dots (iv) \quad (1 \text{ Mark})$$

Substitute equations (iii) and (iv) into equation (i)

$$\tan 3\theta = \frac{\sin 3\theta}{\cos 3\theta} = \frac{3 \sin 3\theta - 4 \sin^3 \theta}{4 \cos^3 \theta - 3 \cos \theta}$$

$$\tan 3\theta = \frac{3 \sin 3\theta - 4 \sin^3 \theta}{4 \cos^3 \theta - 3 \cos \theta}$$

Divide by $\cos^3 \theta$ to each term on right hand side

$$\tan 3\theta = \frac{3 \tan \theta \sec^2 \theta - 4 \tan^3 \theta}{4 - 3 \sec^2 \theta} \text{ but } \sec^2 \theta = 1 + \tan^2 \theta$$

$$\tan 3\theta = \frac{3 \tan \theta (1 + \tan^2 \theta) - 4 \tan^3 \theta}{4 - 3(1 + \tan^2 \theta)} = \frac{3 \tan \theta + 3 \tan^3 \theta - 4 \tan^3 \theta}{4 - 3 - 3 \tan^2 \theta}$$

$$\tan 3\theta = \frac{3 \tan \theta - \tan^3 \theta}{1 - 3 \tan^2 \theta} \text{ but } \tan \theta = t$$

$$\tan 3\theta = \frac{3t - t^3}{1 - 3t^2} \text{ hence shown.} \quad (1 \text{ Mark})$$

Also given, $t^3 - 3t^2 - 3t + 1 = 0$

$$\frac{3t - t^3}{t - 3t^2} = 1$$

$$\text{Comparing with; } \tan 3\theta = \frac{3t - t^3}{t - 3t^2} \quad (0.5 \text{ Mark})$$

$$\tan 3\theta = 1$$

$$3\theta = \tan^{-1}(1) = 45^\circ$$

From general formula of tangent; $3\theta = 180^0n + 45^0$

$$\theta = 15^0 (4n + 1)$$

$$\theta = 15^0, 75^0, 135^0, 195^0, \dots \quad (0.5 \text{ Mark})$$

but; $t = \tan \theta$

$$t_1 = 0.2679, t_2 = 3.7321 \text{ and } t_3 = -1 \quad (1 \text{ Mark})$$

(c) Given $\tan z = 3$

But $\tan = -i \operatorname{tanh}(iz)$

$$-i \operatorname{tanh}(iz) = 3, \operatorname{tanh}(iz) = 3i$$

$$\operatorname{Tanh}(iz) \frac{e^{iz} - e^{-iz}}{e^{iz} + e^{-iz}} = 3i \quad (1 \text{ Mark})$$

$$e^{iz} - e^{-iz} = 3i(e^{iz} + 3ie^{-iz})$$

$$e^{iz} - e^{-iz} = 3ie^{iz} + e^{-iz}$$

$$(1 - 3i) e^{iz} - (1 + 3i)e^{-iz} = 0$$

$$(1 - 3i) e^{2iz} - (1 + 3i) = 0$$

$$e^{2iz} = \frac{1+3i}{1-3i}$$

$$e^{2iz} = \left(\frac{1+3i}{1-3i}\right) \left(\frac{1+3i}{1+3i}\right) = \frac{-4}{5} + \frac{3}{5}i$$

$$2iz = \ln\left(-\frac{4}{5} + \frac{3}{5}i\right)$$

$$z = \frac{1}{2i} \ln\left(-\frac{4}{5} + \frac{3}{5}i\right)$$

$$\therefore z = -\frac{i}{2} \ln\left(-\frac{4}{5} + \frac{3}{5}i\right) \quad (2 \text{ Marks})$$

(d) Proceed as follow;

Given that $|z| = 3$ but $z = x + iy$

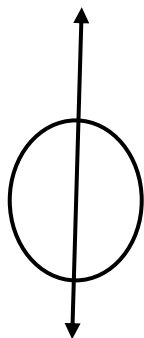
$$|x + iy| = 3$$

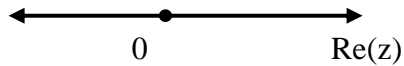
$$\sqrt{x^2 + y^2} = 3$$

$$x^2 + y^2 = 3^2$$

Sketch

Im(z)





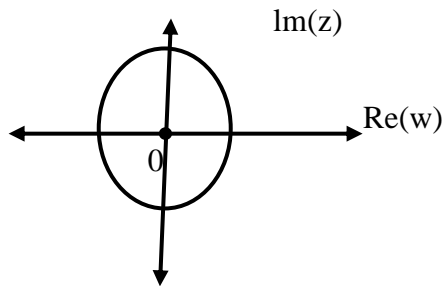
(2 Marks)

Under transformation $|w| = 2 |z|$ but $|z| = 3$ and $w = u + iv$

$$|u + iv| = 2(3)$$

$$\sqrt{u^2 + v^2} = 6$$

$$u^2 + v^2 = 6^2$$



(2 Marks)

The image of the circle is $u^2 + v^2 = 6^2$ with centre at origin and radius 6 units.

SECTION B (40 Marks)

Student should any TWO question from this section.

5. (a) (i) Consider L.H.S

$$\sqrt{\frac{1-\sin\theta}{1+\sin\theta}} = \sqrt{\frac{(1-\sin\theta)(1-\sin\theta)}{(1+\sin\theta)(1-\sin\theta)}} = \sqrt{\frac{(1-\sin\theta)^2}{1-\sin^2\theta}} = \sqrt{\frac{(1-\sin\theta)^2}{\cos^2\theta}} = \frac{1-\sin\theta}{\cos\theta} = \frac{1}{\cos\theta} - \frac{\sin\theta}{\cos\theta}$$

$$\sqrt{\frac{1-\sin\theta}{1+\sin\theta}} = \sec\theta - \tan\theta, \text{ hence shown} \quad (3 \text{ Marks})$$

(ii) Required to eliminate θ

$$x = \sin\theta + \cos\theta \dots\dots\dots (i)$$

$$y = \sin\theta - \cos\theta \dots\dots\dots (ii)$$

add equation (i) and (ii)

$$x + y = 2\sin\theta$$

$$\frac{x+y}{2} = \sin\theta \dots\dots\dots (iii)$$

Subtract equations (i) and (ii)

$$x - y = 2\cos\theta$$

$$\frac{x-y}{2} = \sin\theta \dots\dots\dots (iv)$$

Subtract equation (iii) and (iv) into $\cos^2\theta + \sin^2$

$$\left(\frac{x-y}{2}\right)^2 + \left(\frac{x+y}{2}\right)^2 = 1$$

$$\therefore (x-y)^2 + (x+y)^2 = 4$$

(b) Required the max, and min, value of a function.

Consider $\frac{1}{5\cos x + 12\sin x - 10} = \frac{1}{R\cos(x-\alpha) - 10}$

Express $5\cos x + 12\sin x = R\cos(x - \alpha)$

$$5\cos x + 12\sin x = R(\cos x \cos \alpha + \sin x \sin \alpha)$$

$$5\cos x + 12\sin x = R\cos x \cos \alpha + \sin x \sin \alpha$$

Equate

$$5 = R\cos \alpha \dots\dots\dots (i)$$

$$12 = R\sin \alpha \dots\dots\dots (ii)$$

Square equations (i) and (ii) then add

$$5^2 + 12^2 = R^2\cos^2\alpha + R^2\sin^2\alpha$$

$$169 = R^2(\cos^2\alpha + \sin^2\alpha)$$

$$R^2 = 169$$

$$R = 13 \quad (1\text{Mark})$$

$$5\cos x + 12\sin x = 13 \cos(x-\alpha)$$

Therefore $\frac{1}{5\cos x + 12\sin x - 10} = \frac{1}{13\cos(x-\alpha) - 10}$

At maximum $\cos(x-\alpha) = 1$

$$\frac{1}{5\cos x + 12\sin x - 10} = \frac{1}{13-10} = \frac{1}{3} \quad (1\text{ Mark})$$

At minimum $\cos(x-\alpha) = -1$

$$\frac{1}{5\cos x + 12\sin x - 10} = \frac{1}{-13-10} = \frac{-1}{23} \quad (1\text{Mark})$$

∴ The maximum value is $\frac{1}{3}$ and the minimum value is $\frac{-1}{23}$

(c) Given A, B and C as angles of a triangle

$$\sin^2 A + \sin^2 B + \sin^2 C = 4 \sin A \sin B \sin C$$

Consider LHS

$$\sin^2 A + \sin^2 B + \sin^2 C = \sin(A+B) \cos(A-B) + 2 \sin C \cos C \dots\dots\dots (i)$$

$$\text{From } A + B + C = \pi, A + B = \pi - C$$

$$\sin(A+B) = \sin(\pi - C) \quad (1 \text{Mark})$$

$$\sin(A+B) = \sin C \dots\dots\dots(ii)$$

Substitute equation (ii) into equation (i)

$$\sin^2 A + \sin^2 B + \sin^2 C = 2 \sin C \cos(A-B) + 2 \sin C \cos C$$

$$\sin^2 A + \sin^2 B + \sin^2 C = 2 \sin C [\cos(A-B) + \cos C] \dots\dots\dots(iii)$$

Also apply cos into $A + B = \pi - C$

$$\cos(A+B) = \cos(\pi - C)$$

$$\cos(A+B) = -\cos C \dots\dots\dots (iv)$$

Substitute equation (iv) into equation (iii)

$$\sin^2 A + \sin^2 B + \sin^2 C = 2 \sin C [\cos(A-B) - \cos(A+B)]$$

$$\sin^2 A + \sin^2 B + \sin^2 C = 2 \sin C [-2 \sin A \sin(-B)]$$

$$\therefore \sin^2 A + \sin^2 B + \sin^2 C = 4 \sin A \sin B \sin C \quad (2 \text{Marks})$$

(d) Required to solve for θ

$$\sin 3\theta - \sin \theta = 0$$

$$3 \sin \theta - 4 \sin^3 \theta - \sin \theta = 0$$

$$2 \sin \theta - 4 \sin^3 \theta = 0$$

$$\sin \theta (1 - 2 \sin^2 \theta) = 0$$

$$\sin \theta = 0 \text{ or } \sin \theta = \pm \frac{\sqrt{2}}{2} \quad (2 \text{ Marks})$$

Consider $\sin \theta = 0$

$$\theta = \sin^{-1} 0 = 0^0, (\alpha = 0^0)$$

From $\theta = \pi n + (-1)^n \alpha$ where $n = 0, 1, 2$

$$\theta = 180^0 n$$

$\theta = 0, 180^\circ$ (1 Mark)

Also Consider $\sin\theta = \frac{\sqrt{2}}{2}$

$\theta = \sin^{-1} \frac{\sqrt{2}}{2} = 45^\circ, (\alpha = 45^\circ)$

From $\theta = \pi n + (-1)^n \alpha$ where $n = 0, 1, 2$

$\theta = 180^\circ n - (-1)^n 45^\circ$

$\theta = 45^\circ, 135^\circ$ (1 Mark)

Also consider $\sin\theta = \frac{\sqrt{2}}{2}$

$\theta = \sin^{-1} \left(-\frac{\sqrt{2}}{2} \right) = -45^\circ, (\alpha = -45^\circ)$

From $\theta = \pi n + (-1)^n \alpha$ where $n = 0, 1, 2, \dots$

$\theta = 180^\circ n + (-1)^n (-45^\circ)$

$\theta = 225^\circ, 315^\circ$ (1Mark)

6. (a) (i) Given; $\frac{1}{x^3-1}$

$\frac{1}{x^3-1} = \frac{1}{(x-1)(x^2+x+1)}$ (i)

$\frac{1}{(x-1)(x^2+x+1)} = \frac{A}{x-1} + \frac{Bx+C}{x^2+x+1}$

$1 = A(x^2 + x + 1) + (x - 1)(Bx + C)$

$1 = A(x^2 + x + 1) + Bx^2 + (C - B)x - C$

Equate

$0 = A + B$ (ii)

$0 = A - B + C$ (iii)

$1 = A - C$ (iv)

From equation (iv) $C = A - 1$ Substitute into equation (iii)

$0 = A - B + A - 1$

$2A - B = 1$ (v)

Solve equations (ii) and (v) simultaneously

$A + B = 0$

$$2A - B = 1$$

$$A = \frac{1}{3}, B = \frac{-1}{3}, C = A - 1 = -\frac{2}{3}$$

$$\therefore \frac{1}{x^3-1} = \frac{1}{3(x-1)} - \frac{x+2}{3(x^2+x+1)}$$

(2 Marks)

(ii) Assume α, β and γ be the roots of a cubic equation.

$$\text{Sum of roots} = \alpha + \beta + \gamma = 6 \dots\dots\dots (i)$$

$$(\alpha + \beta + \gamma)^2 = \alpha^2 + \beta^2 + \gamma^2 + 2(\alpha\beta + \alpha\gamma + \beta\gamma)$$

$$6^2 = 14 + 2(\alpha\beta + \alpha\gamma + \beta\gamma)$$

$$2(\alpha\beta + \alpha\gamma + \beta\gamma) = 22$$

$$\text{Sum of product of roots} = \alpha\beta + \alpha\gamma + \beta\gamma = 11 \dots\dots\dots (ii) \quad (1.5 \text{ Marks})$$

$$(\alpha + \beta + \gamma)^3 = (\alpha^3 + \beta^3 + \gamma^3 + 6\alpha\beta\gamma + 3\alpha\beta(\alpha + \beta) + 3\alpha\gamma(\alpha + \gamma) + 3\beta\gamma(\beta + \gamma))$$

$$6^3 = (36 + 6\beta\gamma + 3\alpha\beta(6-\gamma) + 3\alpha\gamma(6-\beta) + 3\beta\gamma(6-\alpha))$$

$$216 = 36 + 6\alpha\beta\gamma + 18(\alpha\beta + \alpha\gamma + \beta\gamma) - 9\alpha\beta\gamma$$

$$180 = 18(11) - 3\alpha\beta\gamma$$

$$\alpha\beta\gamma = 6$$

$$\text{Product of roots} = \alpha\beta\gamma = 6 \dots\dots\dots (iii) \quad (1.5 \text{ Marks})$$

Now recall the cubic equation;

$$X^3 - (\text{sum of roots})x^2 + (\text{Sum of product of root})x - (\text{product of roots}) = 0$$

$$X^3 - 6x^2 + 11x - 6 = 0 \quad (0.5 \text{ Mark})$$

$$\alpha = 3, \beta = 1 \text{ and } \gamma = 2 \quad (1.5 \text{ Marks})$$

(b) Required to show that; $\sum_{r=1}^n r^2 = \frac{n}{6}(n+1)(2n+1)$

Consider the series below

$$1^2 + 2^2 + 3^2 + 4^2 + 5^2 + \dots + n^2$$

In sigma notation the above series can be written as

$$1^2 + 2^2 + 3^2 + 4^2 + \dots + n^2 = \sum_{r=1}^n r^2 \quad (1 \text{ Mark})$$

$$\text{Now consider } \sum_{r=1}^n (r+1)^3 - r^3$$

$$\text{When } r = 1, 2^3 - 1^3$$

$$\text{When } r = 2, 3^3 - 2^3$$

$$\text{When } r = 3, 4^3 - 3^3$$

When $r = n - 2$, $(n - 1)^3 - (n - 2)^3$

When $r = n - 1$ $(n^3 - (n-1)^3$

When $r = n$, $(n+1)^3 - n^3$

$$\sum_{r=1}^n (r + 1)^3 - r^3 = (n + 1)^3 - 1^3$$

$$\sum_{r=1}^n (r^3 + 3r^2 + 3r + 1 - r^3) = (n + 1)^3 - 1^3$$

$$\sum_{r=1}^n (3r^2 + 3r + 1) = (n + 1)^3 - 1$$

$$\sum_{r=1}^n (3r^2 + \sum_{r=1}^n 3r + \sum_{r=1}^n 1) = (n + 1)^3 - 1$$

$$3\sum_{r=1}^n r^2 + 3\sum_{r=1}^n r + \sum_{r=1}^n 1 = (n + 1)^3 - 1$$

$$3\sum_{r=1}^n r^2 + 3\left(\frac{n}{2}(n + 1)\right) + n = (n + 1)^3 - 1$$

$$3\sum_{r=1}^n r^2 = (n + 1)^3 - 1 - \frac{3n}{2}(n + 1) - n$$

$$3\sum_{r=1}^n r^2 = (n + 1)^3 - \left(\frac{3n}{2}(n + 1) + (n + 1)\right)$$

$$3\sum_{r=1}^n r^2 = \frac{2(n+1)^3 - 3n(n+1) - 2(n+1)}{2}$$

$$3\sum_{r=1}^n r^2 = \left(\frac{n+1}{2}\right) (2(n+1)^2 - 3n - 2)$$

$$3\sum_{r=1}^n r^2 = \left(\frac{n+1}{2}\right) (2(n^2 + 4n + 2) - 3n - 2)$$

$$3\sum_{r=1}^n r^2 = \left(\frac{n}{6}\right) (n+1) (2n+1)$$

(1 Mark)

Now for, $\sum_{r=1}^n (r - 3)^2$

$$3\sum_{r=1}^n (r - 3)^2 = \sum_{r=1}^n (r^2 - 6r + 9) = \sum_{r=1}^n r^2 - 6\sum_{r=1}^n r + \sum_{r=1}^n 9$$

$$3 \sum_{r=1}^n (r-3)^2 = \frac{n}{6} (n+1)(2n+1) - 6 \left(\frac{n}{2} (n+1) \right) + 9n$$

$$3 \sum_{r=1}^n r^2 = (r-3)^2 = \frac{n}{6} (n+1)(n+1) - 3n(n+1) + 9n$$

$$3 \sum_{r=1}^n r^2 = (r-3)^2 = \frac{n}{6} (n+1)(2n+1) - 3n^2 + 6n = \frac{n}{6} ((n+1)(2n+1) - 18n + 36)$$

$$\therefore 3 \sum_{r=1}^n r^2 = (r-3)^2 = \frac{n}{6} (2n^2 - 15n + 37) \quad (1 \text{ Mark})$$

$r=1$

(c) Proceed as follows;

$$\text{Consider } \frac{2x^2 - 7x - 4}{3x^2 - 14x + 11} - 2 > 0$$

$$\frac{-4x^2 + 21x - 26}{3x^2 - 14x + 11} > 0$$

$$\frac{4x^2 - 21x + 26}{3x^2 - 14x + 11} < 0$$

$$\frac{(4x - 13)(x - 2)}{(3x - 11)(x - 1)} < 0$$

$$\text{Boundaries } x = 1, x = 2, x = \frac{13}{4}, x = \frac{11}{3}$$

(2 Marks)

	$x < 1$	$1 < x < 2$	$2 < x < \frac{13}{4}$	$\frac{13}{4} < x < \frac{11}{3}$	$x > \frac{11}{3}$
$4x - 13$	- ve	- ve	- ve	+ ve	+ ve
$x - 2$	- ve	- ve	+ ve	+ ve	+ ve
$(4x - 13)(x - 2)$	+ ve	+ ve	- ve	+ ve	+ ve
$(3x - 11)$	- ve	- ve	- ve	- ve	+ ve
$(x - 1)$	- ve	+ ve	+ ve	+ ve	+ ve
$(3x - 11)(x - 1)$	+ ve	- ve	- ve	- ve	+ ve
$(4x - 13)(x - 2)$	+ ve	- ve	+ ve	- ve	+ ve

$(3x - 11)(x - 1)$					
--------------------	--	--	--	--	--

$\therefore 1 < x < 2$ and $\frac{13}{4} < x < \frac{11}{3}$ (1 Mark)

(d) Proceed as follows

Given proposition $\frac{d}{dx}(x^n) = nx^{n-1}$

Step1: We need to prove if it is true for $n = 1, 2$

when $n = 1$, $\frac{d}{dx}(x^1) = 1x^{1-1}$

$1 = 1$ it is true for $n = 1$ (i) (1 Mark)

when $n = 2$, $\frac{d}{dx}(x^2) = 2x^{2-1}$

$2x = 2x$ it true for $n = 2$ (ii)

Step2: Assume it is true for $n = k$

$\frac{d}{dx}(x^k) = kx^{k-1}$ (iii) (1Mark)

Step 3: We need to prove if it is true for $n = k + 1$

$\frac{d}{dx}(x^{k+1}) = (k+1)x^k$

Consider L.H.S and then prove if it is equal to R.H.S

$\frac{d}{dx}(x^{k-1}) = \frac{d}{dx}(xx^k) = x^k \frac{d}{dx}(x) + \frac{d}{dx}(x^k)$

$\frac{d}{dx}(x^{k+1}) = x^k + \frac{d}{dx}(x^k)$ but $\frac{d}{dx}(x^k) = kx^{k-1}$ from equation (iii)

$\frac{d}{dx}(x^{k+1}) = x^k + x [kx^{k-1}] = x^k + kx^k = (1+k)x^k = (k+1)x^{k+1-1}$

$\frac{d}{dx}(x^{k+1}) = (k+1)x^{(k+1)-1}$ but $n = k + 1$

$\therefore \frac{d}{dx}(x^n) = nx^{n-1}$ (1 Mark)

7. (a) Required to form a D.E

$y = x^2 + Ae^{2x} + Be^{3x}$ (i)

$y' = 2x + 2Ae^{2x} + 3Be^{3x}$ (ii)

$y'' = 2 + 4Ae^{2x} + 9Be^{3x}$ (iii)

From equation (i)

$$y - x^2 - Be^{3x} = Ae^{2x} \dots\dots\dots (iv)$$

Substitute equation (iv) into equation (ii)

$$y' = 2x + 2(y - x^2 - Be^{3x}) + 3Be^{3x}$$

$$y' - 2x - 2y + 2x^2 = Be^{3x} \dots\dots\dots (v) \quad (0.5 \text{ Mark})$$

Substitute equation (iv) into equation (iii)

$$y'' = 2 + 4(y - x^2 - Be^{3x}) + 9Be^{3x}$$

$$y'' - 2 - 4y + 4x^2 = 5Be^{3x} \dots\dots\dots (vi) \quad (0.5 \text{ Mark})$$

Substitute equation (v) into equation (vi)

$$y'' - 2 - 4y + 4x^2 = 5(y' - 2x - 2y + 2x^2) \dots\dots\dots (vi) \quad (0.5 \text{ Mark})$$

$$\therefore y'' - 5y' + 6y = 6x^2 - 10x + 2 \dots\dots\dots (2 \text{ Marks})$$

(b) Solving a D.E

$$\frac{dy}{dx} = \frac{x-y+2}{x+y} \dots\dots\dots (i)$$

Let $x = X + h, y = Y + k$ and $\frac{dy}{dx} = \frac{dY}{dX} \dots\dots\dots (ii)$

Substitute equation (ii) into equation (i)

$$\frac{dY}{dX} = \frac{(X+h)-(Y+k)+2}{(X+h)+(Y+k)}$$

$$\frac{dY}{dX} = \frac{(X-Y+h-k+2)}{X+Y+h+k} \dots\dots\dots (iii)$$

let and solve

$$h - k + 2 = 0$$

$$h + k = 0$$

$$h = -1, k = 1$$

Then equation (iii) change to

$$\frac{dY}{dX} = \frac{X-Y}{X+Y} \text{ (This is homogeneous D.E) } \dots\dots\dots (iv) \quad (1 \text{ Mark})$$

Let $Y = UX, \frac{dY}{dX} = U + X \frac{dU}{dX} \dots\dots\dots (v) \quad (1 \text{ Mark})$

Substitute equation (v) into equation (iv)

$$U + X \frac{dU}{dX} = \frac{X-UX}{X+UX}$$

$$U + X \frac{dU}{dX} = \frac{1-U}{1+U}$$

$$X \frac{dU}{dX} = \frac{1-U}{1+U} - U$$

$$X \frac{dU}{dX} = \frac{1-2U-U^2}{1+U}$$

$$\int \frac{1+U}{1-2U-U^2} dU = \int \frac{1}{X} dx$$

$$-\frac{1}{2} \ln(1 - 2U - U^2) = \ln AX$$

$$\ln \left(1 - 2\frac{Y}{X} - \frac{Y^2}{X^2} \right) = \ln (AX)^{-1}$$

$$\left(\frac{X^2 - 2XY - Y^2}{X^2} \right) = \frac{C}{X^2}$$

$$X^2 - 2XY - Y^2 = C \dots\dots\dots(vi) \quad (1\text{Mark})$$

But $X = x - h$, $Y = y - k$ and $h = -1$, $k = 1$

i.e $X = x + 1$, $Y = y - 1$ (vii)

substitute equation (vii) into equation (vi)

$$(x+1)^2 - 2(x+1)(y-1) - (y-1)^2 = C \quad (1\text{ Mark})$$

$$\therefore x^2 - y^2 + 4x - 2xy + 2 = C$$

Given; $y = 1$ when $x = 0$

on substituting the values of x and y into the equation; $C = -1$ (0.5 Mark)

$$x^2 - y^2 + 4x - 2xy + 3 = 0$$

(c) Solving $\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + y = 6e^x$.

$$y'' - 2y' + y = 0$$

Write it auxiliary Quadratic Equation (A.Q.E)

$$m^2 - 2m + 1 = 0$$

$$m = 1$$

$$\text{let } \alpha = \beta = p = 1$$

From general solution D.E with two distinct roots

$$y = (Ax + B)e^{px}$$

$$\therefore y_{cf} = (Ax + B)e^x \text{ (Complimentary solution)} \quad (I\text{ Mark})$$

Also consider Particular Integral Part, since e^x and xe^x is contained then,

$$y = kx^2e^x \dots\dots\dots(ii)$$

$$y' = kx^2e^x + 2kxe^x \dots\dots\dots (iii) \quad (1 \text{ Mark})$$

$$y'' = kx^2e^x + 4kxe^x + 2ke^x \dots\dots\dots (iv)$$

Substitute equations (ii), (iii) and (iv) into equation (i)

$$(kx^2e^x + 4kxe^x + 2ke^x) - 2(kx^2e^x + 2kxe^x) + 2ke^x = 6e^x$$

$$2ke^x = 6e^x$$

Equate L.H.S and R.H.S to obtain the value k

$$k=3$$

$$\therefore y_{PI} = 3x^2e^x \text{ (Particular Integral Solution)} \quad (1 \text{ Mark})$$

Therefore, $e_y = y_{cf} + y_{p.I}$ (General Solution)

$$\therefore y = (Ax + B)e^x + \underline{3x^2e^x} \quad (2 \text{ Marks})$$

(d) Proceed as follows

$$6 \frac{d^2y}{dt^2} + \frac{dy}{dt} = 0 \dots\dots\dots (i)$$

$$z = \frac{dy}{dt}, \frac{d^2z}{dt^2} = \frac{d^2y}{dt^2} \quad (1 \text{ Mark})$$

$$6 \frac{dz}{dt} + z = 0 \text{ (This separable D.E)}$$

$$6 \int \frac{1}{z} dz = - \int dt$$

$$6 \ln z = -t + A$$

$$6 \ln z = -\frac{t}{6} + B$$

$$Z = e^{\left(-\frac{t}{6} + B\right)} = e^{\left(-\frac{t}{6}\right)} e^B = Ce^{\left(-\frac{t}{6}\right)}$$

$$\frac{dy}{dt} = Ce^{\left(-\frac{t}{6}\right)} \quad (1 \text{ Mark})$$

$$\int dy = C \int e^{\frac{-t}{6}} dt$$

$$y = -6Ce^{\left(\frac{-t}{6}\right)} + D$$

when $y = 63, t = 0$

$$63 = -6C + D \dots\dots\dots (i)$$

When $y = 36, t = 6 \ln 4$

$$72 = -3C + 2D \dots\dots\dots (ii)$$

Solve equation (i) and equation (ii) simultaneously

$$\begin{cases} 63 = -6C + D \\ 72 = -3C + 2D \end{cases}$$

$$C = -6, D = 27$$

$$\text{Then from } y = -6e^{\left(\frac{-t}{6}\right)} + D$$

$$\therefore y = 36e^{\left(\frac{-t}{6}\right)} + 27$$

$$y = 36e^{\left(\frac{-t}{6}\right)} + 27$$

$$\frac{dy}{dt} = -6e^{\left(\frac{-t}{6}\right)} \text{ but } \frac{dy}{dt} \text{ is rate of cooling must be negative}$$

$$\frac{dy}{dt} = 6e^{\left(\frac{-t}{6}\right)} \text{ Given that } \frac{dy}{dt} < 1$$

$$6e^{\left(\frac{-t}{6}\right)} = \frac{dy}{dt} < 1$$

(1Mark)

$$6e^{\left(\frac{-t}{6}\right)} < 1$$

$$t < -6 \ln \frac{1}{6}$$

$$t < 10.75$$

The rate of cooling of the body will have fallen below one degree per minute after 10 minutes.

8. (a) proceed as follows;

$$4x^2 + 25y^2 - 24x + 50y - 39 = 0$$

$$4x^2 - 24x + 25y^2 + 50y = 39$$

$$4(x^2 - 6x) + 25(y^2 + 2y) = 39 \text{ (by completing the square)}$$

$$4(x-3)^2 + 25(y+1)^2 = 100$$

$$\frac{(x-3)^2}{25} + \frac{(y+1)^2}{4} = 1 \text{ compare with } \frac{(x-h)^2}{a^2} + \frac{(y+k)^2}{b^2} \quad (1 \text{ Mark})$$

$$a^2 = 25, a = 5$$

$$b^2 = 4, b = 2$$

$$h = 3, k = -1$$

\therefore The centre of an ellipse is (3, -1) (1Mark)

$$\text{Since } a > b \text{ then } a^2(1 - e^2) = b^2$$

$$e = \sqrt{1 - \frac{b^2}{a^2}} = \sqrt{1 - \frac{4}{25}} = \frac{\sqrt{21}}{5}$$

∴ Eccentricity is $\frac{\sqrt{21}}{5}$

(1 Mark)

∴ Foci are $(\pm ae+h,k) = (\pm\sqrt{21} + 3, -1)$

(1 Mark)

∴ Equation of directrices are $x = \pm \frac{a}{e} + h = \pm \frac{25}{\sqrt{21}} + 3$

(1 Mark)

(b) Required, to show a condition for a line $lx + my = n$ to touch a hyperbola $xy = C^2$

Substitute a line $lx + my = n$ into hyperbola $xy = C^2$

$$x \left(\frac{n-lx}{m} \right) = C^2$$

(1 Mark)

$$nx - lx^2 = mc^2$$

$$lx^2 - nx + mc^2 = 0$$

Check if it satisfy perfect square (i.e condition for tangency, $b^2 = 4ac$)

$$b^2 = n^2$$

$$4ac = 4mlc^2$$

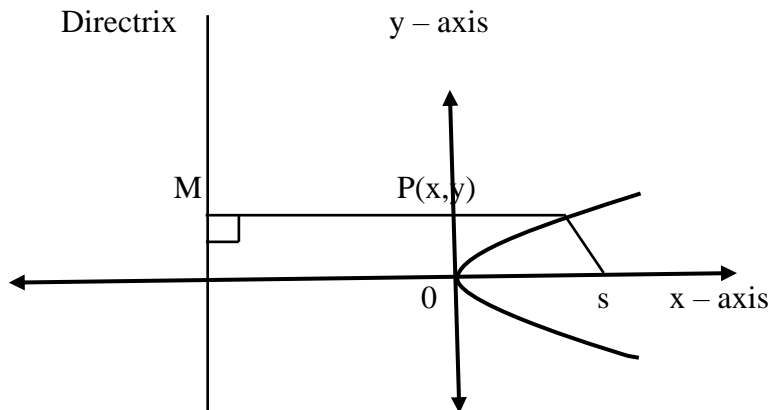
$$\text{But } n^2 = 4mlc^2$$

(2 Marks)

∴ The given line is a tangent to the rectangular hyperbola

(c) (i) Parabola is a conic sections whose eccentricity is one ($e = 1$)

(1 Mark)



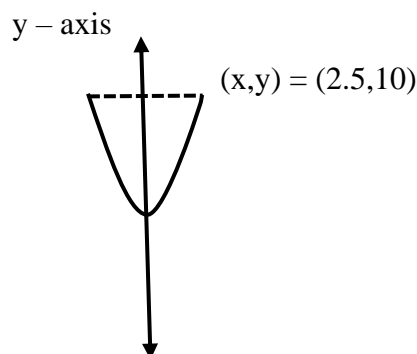
(1.5 Marks)

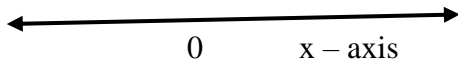
Where; $e = \frac{SP}{MP}$

(0.5 Mark)

(ii) Proceed as follows;

Consider the figure below





(1Mark)

From $r^2 = 4by$ at $(x,y) = (2.5,10)$

$$(2.5)^2 = 4b(10), b = \frac{5}{32}$$

Then $x^2 = \frac{5}{8}y$

(1Mark)

At $y = 2, x = \frac{\sqrt{5}}{2}$

\therefore Wide of an arch = $2x = 2\left(\frac{\sqrt{5}}{2}\right) = \sqrt{5}$

(2Marks)

(e) Proceed as follows;

Table of value $r = 3\cos 2\theta$

0°	30°	60°	90°	120°	150°	180°	210°	240°	270°	300°	330°	360°
0	$\frac{\pi}{6}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$\frac{2\pi}{3}$	$\frac{5\pi}{6}$	π	$\frac{7\pi}{6}$	$\frac{4\pi}{3}$	$\frac{3\pi}{2}$	$\frac{5\pi}{3}$	$\frac{11\pi}{6}$	2π
3	1.5	-1.5	-3	-1.5	1.5	3	1.5	-1.5	-3	-1.5	1.5	3

(2Marks)

The curve of $r = 3\cos 2\theta$

